Proceedings of the Jangjeon Mathematical Society
27 (2024), No. 4, pp. 809 - 816

www.jangjeon.or.kr
http://dx.doi.org/10.17777/pjms2024.27.4.809

STATISTICAL CONVERGENCE OF SEQUENCE IN NON-ARCHIMEDEAN FUZZY METRIC SPACE BY NÖRLUND-(M, λ_n) AND RIESZ MEAN (M, λ_n) METHOD OF SUMMABILITY

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Abstract. Let K denotes a complete non-trivially valued non-Archimedean field. In this paper, we define statistical convergence and statistical summability of fuzzy sequences by Nörlund-(M, λ_n) and Riesz mean (M, λ_n) method of summability. Further, discussed inclusion relation between statistical convergence and statistical summability of such sequences.

2020 Mathematics Subject Classification. 26E30, 40A35,40G05, 40G15.

Keywords and phrases. Nörlund-(M, λ_n) summability method, Riesz mean (M, λ_n) summability method, Statistical convergence, Non-Archimedean field, Non-Archimedean fuzzy metric space, Statistically summable.

1. Introduction

In non-Archimedean analysis the study focuses on leveraging the power of ultrametric property, which also satisfies the triangular inequality of metric spaces. Kurt Hensel [8] defined the theory of p-adic fields, which are non-Archimedean in nature. Fast [4] introduced the concept of statistical convergence for a sequence of reals. Fast introduced an extension of usual concept of sequential limits, called statistical convergence. Despite being independent of one another, Steinhaus [22] and Fridy [5] initiated the research of statistical convergence. Statistical convergence for real and complex sequences was independently developed by Buck [2, 3] and Schoenberg [21]. Fridy found a criteria for statistically convergent real sequences analogous to cauchy criteria of convergence. For sequences in any Hausdroff locally compact topological vector space, Maddox [9] extended statistical convergence. Statistically convergent and statistically Cauchy sequences for reals were examined by Rath and Tripathy [18]. Suja and Srinivasan [23] introduced statistical convergence in ultrametric field. Muthu and Suja [12, 13] investigated weighted and generalized Nörlund-Euler statistical convergence in ultrametric field.

The theory of summability has many uses throughout analysis and applied mathematics. Euler, Gauss, Cauchy and Abel contributed to the development of summability theory, and there are special methods of summability such as Abel, Borel, Euler, Taylor, Nörlund, Hausdroff in classical analysis. Natarajan [14] introduced (M,λ_n) method of summability in ultrametric fields.

In 1965, Zadeh [28] introduced the notion of fuzzy sets. George and Veeramani [7] defined fuzzy metric space. Matloka [10] introduced convergence of sequences of fuzzy numbers in reals. In 1995, Nuray and Savas [15] defined statistical convergence of fuzzy numbers. Tripathy and Baruah [26] studied some properties of Nörlund and Riesz mean of sequences of fuzzy real numbers. Pattanaik et al.

[17] investigated regularity properties of Nörlund and Nörlund-type means for sequence of fuzzy numbers. Talo and Bal [25] introduced statistical summability $(\overline{N}, P)$ of sequences of fuzzy numbers. Paknazar [16] investigated new class of proximal contraction mappings and established best proximity point theorems. Mihet [11] introduced fuzzy ψ contractive mappings in non-Archimedean fuzzy metric space. The issue of determining the optimal proximity point of the shortest distance between two nonempty sets in a non-Archimedean fuzzy metric space was studied by Vetro and Salimi [27].

More authors [19, 20, 24] have been discussed about the fuzzy numbers in Archimedean analysis. In this article, we introduce some new results of statistical convergence and statistical summability of fuzzy sequences by Nörlund-(M, λ_n) and Riesz mean (M, λ_n) method of summability in non-Archimedean analysis. We establish the inclusion relation between statistical convergence and statistical summability of fuzzy sequences by Nörlund-(M, λ_n) and Riesz mean (M, λ_n) method of summability over non-Archimedean fields.

For a general reference on Ultrametric analysis, the book is [1]. Throughout this article, K denotes a complete nontrivially valued non-Archimedean field.

2. Preliminaries

In this section, some preliminary definitions of sequence in fuzzy metric space, statistical convergence, statistical summability of non-Archimedean fuzzy metric space using Nörlund and Riesz mean (or Nörlund-type transformation) methods of summability are defined.

Statistical convergence of sequence $x = \{x_n\}$ and statistical summability in non-Archimedean fuzzy metric space using Nörlund-(M, λ_n) and Riesz mean (or Nörlund-type transformation) (M, λ_n) methods of summability are defined. Also, inclusion relation between statistical convergence and statistical summability of non-Archimedean fuzzy metric space by Nörlund-(M, λ_n) and Riesz mean (or Nörlund-type transformation) (M, λ_n) methods of summability are discussed in non-Archimedean fields.

Definition 2.1. A binary operation $* : [0,1] \times [0,1] \to [0,1]$ is called a continuous t-norm if it satisfies the following conditions:
i. $*$ is commutative and associative ;
ii. $*$ is continuous ;
iii. $a * 1 = a \; \forall \; a \in [0,1]$;
iv. $a * b \leq c * d$ when $a \leq c$ and $b \leq d$, with $a, b, c, d \in [0,1]$

Definition 2.2. [7] A fuzzy metric space is an ordered triple $(X, M, *)$ such that X denotes a non-empty set. $*$ refers to a continuous t-norm and M serves as a fuzzy set on $X \times X \times (0, +\infty)$ satisfying the following conditions, for all $x, y, z \in X$ and $t, s > 0$
i. $M(x, y, t) > 0$;
ii. $M(x, y, t) = 1$ if and only if $x = y$;
iii. $M(x, y, t) = M(y, x, t)$;
iv. $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$;
v. $M(x, y, .) : (0, +\infty) \to (0, 1]$ is left continuous
If we replace iv by vi,
vi. $M(x, y, t) * M(y, z, s) \leq M(x, z, max\{t, s\})$;

then the triple $(X, M, *)$ is called non-Archimedean fuzzy metric space. Note that, since vi implies iv, each non-archimedean fuzzy metric space is a fuzzy metric space.

Definition 2.3. [1] A non-Archimedean valuation (or a valued function) is a function $|\ | : X \to \mathcal{R}$ satisfying the following axioms
i. $|x| > 0, |x| = 0$ if and only if $x = 0.\ \forall\ x \in X.$
ii. $|xy| = |x||y|,\ \forall\ x, y \in X.$
iii. $|x + y| = max\{|x|, |y|\},\ \forall\ x, y \in X.$
Example:- p-adic valuation is an example for non-Archimedean valuation.

Definition 2.4. [7] A sequence $\{x_n\}$ in fuzzy metric space $(X, M, *)$ is said to be convergent to a limit ℓ if for any $\varepsilon > 0$, we have $\lim\limits_{n\to\infty} M(x_n, \ell, t) = 1.$

Definition 2.5. A sequence $\{x_n\}$ in fuzzy metric space $(X, M, *)$ is said to be statistically convergent to a limit ℓ if for any $\varepsilon > 0$, we have
$\lim\limits_{n\to\infty} \frac{1}{n}|\{k \le n : M(x_k, \ell, t) \ge 1 - \varepsilon\}| = 0.$

Definition 2.6. A sequence $\{x_n\}$ is said to be summable by Nörlund mean (N, p_n) to L if, $\lim\limits_{n\to\infty} M(\frac{1}{P_n} \sum\limits_{i=1}^{n} p_{n-i+1}x_i, L, t) = 1.$

Definition 2.7. A sequence $\{x_n\}$ is said to be summable to L by Nörlund-(M, λ_n) method if, $\lim\limits_{n\to\infty} M(\sum_{i=0}^{k} \frac{p_i x_i}{P_i} \lambda_k, L, t) = 1.$

Definition 2.8. A sequence $x = \{x_n\}$ is said to be statistically summable to L by Nörlund-(M, λ_n) method, if for any $\varepsilon > 0$,
$\lim\limits_{n\to\infty} \frac{1}{n}\left|\left\{k \le n : M(\sum_{i=0}^{k} \frac{p_i x_i}{P_i} \lambda_k, L, t) \ge 1 - \varepsilon\right\}\right| = 0.$

Definition 2.9. A sequence $\{x_n\}$ is said to be summable by Riesz mean or Nörlund-type transformation to L if, $\lim\limits_{n\to\infty} M(\frac{1}{P_n} \sum\limits_{i=1}^{n} p_i x_i, L, t) = 1.$

Definition 2.10. A sequence $\{x_n\}$ is said to be summable by Nörlund-type (M, λ_n) or Riesz mean (M, λ_n) method to L if, $\lim\limits_{n\to\infty} M(\sum\limits_{k=1}^{n} (\sum\limits_{i=0}^{n-k} \frac{p_{n-i}\lambda_{n-k-i}}{P_i}) x_k, L, t) = 1.$

Definition 2.11. A sequence $x = \{x_n\}$ is said to be statistically summable to L by Nörlund-type (M, λ_n) or Riesz mean (M, λ_n) method, if for any $\varepsilon > 0$,
$\lim\limits_{n\to\infty} \frac{1}{n}\left|\left\{k \le n : M(\sum\limits_{k=1}^{n} (\sum\limits_{i=0}^{n-k} \frac{p_{n-i}\lambda_{n-k-i}}{P_i}) x_k, L, t) \ge 1 - \varepsilon\right\}\right| = 0.$

3. Main Results

Theorem 3.1. *If the sequence $\{x_n\}$ is convergent to L, then it is statistically convergent to L. That is, if $x = \{x_n\}$ is a fuzzy sequence such that $\lim\limits_{n\to\infty} M(x_n, L, t) = 1$, then $\lim\limits_{n\to\infty} |\{k \in N : M(x_k, L, t) \ge 1 - \varepsilon\}| = 0.$*

Proof. Given the sequence $\{x_n\}$ is convergent to L.
That is $\lim\limits_{n\to\infty} M(x_n, L, t) = 1$
Given $\varepsilon > 0,\ \exists\ N \in \mathcal{N}$ such that $M(x_n, L, t) - 1 < \varepsilon\ \forall\ n \ge N$

implies $M(x_n, L, t) - 1 > \varepsilon \; \forall \; n \leq \; N-1$
implies $M(x_n, L, t) > 1 - \varepsilon \; \forall \; n \leq \; N-1$
implies $\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : M(x_k, L, t) \geq 1 - \varepsilon\}| = 0.$ □

Theorem 3.2. *If the sequence $\{x_n\}$ is statistically convergent to L, then it is statistically summable to L by Nörlund-(M, λ_n) method.*

Proof. Let $\{x_n\}$ be a sequence in non-Archimedean fuzzy metric space statistically convergent to L.
$\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : M(x_k, L, t) \geq 1 - \varepsilon\}| = 0.$
To prove $\lim_{n\to\infty} \frac{1}{n} \left|\{k \leq n : \sum_{i=0}^{k} M(\frac{p_i x_i}{P_i}\lambda_k, L, t) \geq 1 - \varepsilon\}\right| = 0.$
Consider, $M(\sum_{i=0}^{k} \frac{p_i x_i}{P_i}\lambda_k, L, t) = \sum_{i=0}^{k} \frac{p_i \lambda_k}{P_i} M(x_i, \frac{P_i}{p_i\lambda_k}L, \frac{P_i}{p_i\lambda_k}t)$
Since $P_i = p_0 + p_1 + \ldots + p_i$,
we have $M(x_i, \frac{P_i}{p_i\lambda_k}L, \frac{P_i}{p_i\lambda_k}t) \geq M(x_i, \frac{P_i}{p_i\lambda_k}L, \frac{max\{p_0, p_1, \ldots, p_i\}}{p_i\lambda_k}t)$

$$M(\sum_{i=0}^{k} \frac{p_i x_i}{P_i}\lambda_k, L, t) \geq M(x_i, \frac{P_i}{p_i\lambda_k}L, \frac{P_i}{p_i\lambda_k}t) \geq M(x_i, \frac{P_i}{p_i\lambda_k}L, \frac{max\{p_0, p_1, \ldots, p_i\}}{p_i\lambda_k}t)$$

Letting limit as $k \to \infty$ in the above equation, we get

$$\begin{aligned}
\lim_{k\to\infty} M(\sum_{i=0}^{k} \frac{p_i x_i}{P_i}\lambda_k, L, t) &\geq \lim_{k\to\infty} M(x_i, \frac{P_i}{p_i\lambda_k}L, \frac{P_i}{p_i\lambda_k}t) \\
&\geq \lim_{k\to\infty} M(x_i, \frac{P_i}{p_i\lambda_k}L, \frac{max\{p_0, p_1, \ldots, p_i\}}{p_i\lambda_k}t) \\
M(\sum_{i=0}^{\infty} \frac{p_i x_i}{P_i}\lambda_k, L, t) &\geq \lim_{k\to\infty} M(x_i, \frac{P_i}{p_i\lambda_k}L, \frac{max\{p_0, p_1, \ldots, p_i\}}{p_i\lambda_k}t) = 1 - \varepsilon. \\
\sum_{i=0}^{\infty} M(\frac{p_i x_i}{P_i}\lambda_k, L, t) &\geq 1 - \varepsilon
\end{aligned}$$

$\lim_{n\to\infty} \frac{1}{n} \left|\{k \leq n : \sum_{i=0}^{k} M(\frac{p_i x_i}{P_i}\lambda_k, L, t) \geq 1 - \varepsilon\}\right| = 0.$ □

Theorem 3.3. *If the sequence $\{x_n\}$ is statistically summable to L by Nörlund-(M, λ_n) method, then it is statistically convergent to L.*

Proof. Suppose

$$\lim_{n\to\infty} \frac{1}{n} \left| \left\{ k \leq n : M(\sum_{i=0}^{k} \frac{p_i x_i}{P_i}\lambda_k, L, t) \geq 1 - \varepsilon \right\} \right| = 0.$$

Then $\exists \; n_1 \in \mathcal{N}$ such that

$$M(\sum_{i=0}^{k} \frac{p_i x_i}{P_i}\lambda_k, L, t) \leq 1 - \varepsilon \; \forall \;\; n_1 \leq k \leq n$$

choose $\{\lambda_k\} = 1, 1, 1, \ldots,$ and $p_k = 1, p_i = 0 \; \forall i \neq k, \; n_1 \leq k \leq n$
we get $|M(x_k, L, t)| \leq 1 - \varepsilon \; \forall \; n_1 \leq k \leq n$
implies $\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : M(x_k, L, t) \geq 1 - \varepsilon\}| = 0.$
$\{x_n\}$ is statistically convergent to L. □

Theorem 3.4. *If the sequence $\{x_n\}$ is statistically summable to L by Riesz mean (M, λ_n) method, then it is statistically convergent to L.*

Proof.

$$\text{Suppose} \lim_{n\to\infty} \frac{1}{n}\left|\left\{k \leq n : M(\sum_{k=1}^{n}(\sum_{i=0}^{n-k} \frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_k, L, t) \geq 1-\varepsilon\right\}\right| = 0.$$

Then $\exists\ n_1 \in \mathcal{N}$ such that

$$M(\sum_{k=1}^{n}(\sum_{i=0}^{n-k} \frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_k, L, t) \leq 1-\varepsilon\ \forall\ n_1 \leq k \leq n$$

choose $\{\lambda_k\} = 1, 1, 1, \ldots$, and $p_k = 1, p_i = 0\ \forall i \neq k,\ n_1 \leq k \leq n$
we get $|M(x_k, L, t)| \leq 1-\varepsilon\ \forall\ n_1 \leq k \leq n$..
implies $\lim\limits_{n\to\infty} \frac{1}{n} |\{k \leq n : M(x_k, L, t) \geq 1-\varepsilon\}| = 0$.
$\{x_n\}$ is statistically convergent to L. □

Theorem 3.5. *If the sequence $\{x_n\}$ is statistically convergent to L, then it is statistically summable to L by Riesz mean (M, λ_n) method.*

Proof. Let $\{x_n\}$ be a sequence in non-Archimedean fuzzy metric space statistically convergent to L.

$$\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : M(x_k, L, t) \geq 1-\varepsilon\}| = 0.$$

$$\text{To prove} \lim_{n\to\infty} \frac{1}{n}\left|\{k \leq n : M(\sum_{k=1}^{n}(\sum_{i=0}^{n-k} \frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_k, L, t) \geq 1-\varepsilon\}\right| = 0.$$

Consider, $M(\sum\limits_{k=1}^{n}(\sum\limits_{i=0}^{n-k} \frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_k, L, t)$

$= M((\sum\limits_{i=0}^{n-1} \frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_1, L, t) + M((\sum\limits_{i=0}^{n-2} \frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_2, L, t)$

$+ \ldots + M((\sum\limits_{i=0}^{n-n-1} \frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_{n-1}, L, t) + M((\sum\limits_{i=0}^{n-n} \frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_n, L, t)$

$= \sum\limits_{i=0}^{n-1} \frac{p_{n-i}\lambda_{n-k-i}}{P_i} M(x_1, \frac{P_i}{\sum_{i=0}^{n-1} p_{n-i}\lambda_{n-k-i}}L, \frac{P_i}{\sum_{i=0}^{n-1} p_{n-i}\lambda_{n-k-i}}t)$

$+ \frac{\sum_{i=0}^{n-2} p_{n-i}\lambda_{n-k-i}}{P_i} M(x_2, \frac{P_i}{\sum_{i=0}^{n-2} p_{n-i}\lambda_{n-k-i}}L, \frac{P_i}{\sum_{i=0}^{n-2} p_{n-i}\lambda_{n-k-i}}t)$

$+ \ldots + \frac{\sum_{i=0}^{n-n-1} p_{n-i}\lambda_{n-k-i}}{P_i} M(x_{n-1}, \frac{P_i}{\sum_{i=0}^{n-n-1} p_{n-i}\lambda_{n-k-i}}L, \frac{P_i}{\sum_{i=0}^{n-n-1} p_{n-i}\lambda_{n-k-i}}t)$

$+ \sum\limits_{i=0}^{n-n} \frac{p_{n-i}\lambda_{n-k-i}}{P_i} M(x_n, \frac{P_i}{\sum_{i=0}^{n-n} p_{n-i}\lambda_{n-k-i}}L, \frac{P_i}{\sum_{i=0}^{n-n} p_{n-i}\lambda_{n-k-i}}t)$

$= \sum_{k=1}^{n} \sum_{i=0}^{n-k} \frac{p_{n-i}\lambda_{n-k-i}}{P_i} M(x_k, \frac{P_i}{\sum_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}L, \frac{P_i}{\sum_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}t)$

Since $P_i = p_0 + p_1 + \ldots + p_i$, we have

$$M(x_k, \frac{P_i}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}L, \frac{P_i}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}t)$$

$$\geq M(x_i, \frac{P_i}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}L, \frac{max\{p_0, p_1, \ldots, p_i\}}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}t) \tag{3.1}$$

Letting limit as $n \to \infty$ in (3.1), we get

$$\lim_{n\to\infty} M(\sum_{k=1}^{n}(\sum_{i=0}^{n-k}\frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_k, L, t)$$

$$\geq \lim_{n\to\infty} M(x_k, \frac{P_i}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}L, \frac{P_i}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}t)$$

$$\geq \lim_{n\to\infty} M(x_k, \frac{P_i}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}L, \frac{max\{p_0, p_1, \ldots, p_i\}}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}t)$$

$$\text{Since } M(\sum_{k=1}^{n}(\sum_{i=0}^{n-k}\frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_k, L, t)$$

$$\geq M(x_k, \frac{P_i}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}L, \frac{max\{p_0, p_1, \ldots, p_i\}}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}t)$$

Further, since $\{x_n\}$ is statistically convergent to L, we have

$$M(\sum_{k=1}^{\infty}(\sum_{i=0}^{n-k}\frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_k, L, t)$$

$$\geq \lim_{n\to\infty} M(\sum_{k=1}^{n}\sum_{i=0}^{n-k}\frac{p_{n-i}\lambda_{n-k-i}}{P_i}x_k, L, t)$$

$$\geq \lim_{n\to\infty} M(x_k, \frac{P_i}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}L, \frac{P_i}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}t)$$

$$\geq \lim_{n\to\infty} M(x_k, \frac{P_i}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}L, \frac{max\{p_0, p_1, \ldots, p_i\}}{\sum\limits_{i=0}^{n-k} p_{n-i}\lambda_{n-k-i}}t) = 1 - \varepsilon.$$

implies $M(\sum\limits_{k=1}^{\infty}(\sum\limits_{i=0}^{n-k}\frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_k, L, t) \geq 1 - \varepsilon$

implies $\lim\limits_{n\to\infty}\frac{1}{n}|\{k \leq n : M(\sum\limits_{k=1}^{n}(\sum\limits_{i=0}^{n-k}\frac{p_{n-i}\lambda_{n-k-i}}{P_i})x_k, L, t) \geq 1 - \varepsilon\}| = 0$.

$\{x_n\}$ is statistically summable to L by Riesz mean (M, λ_n) method. □

4. Conclusion

In this paper, statistical convergence of fuzzy sequences by Nörlund-(M, λ_n) and Riesz mean (or Nörlund-type transformation) (M, λ_n) methods are defined over Non-Archimedean fields. Also, discussed inclusion relation between statistical convergence and statistical summability of fuzzy sequences by Nörlund-(M, λ_n) and Riesz mean (or Nörlund-type transformation) (M, λ_n) methods of summability over Non-Archimedean fields.

5. Acknowledgements

The authors would like to express their thanks to the anonymous reviewers for their valuable feedback and suggestions to improve the quality of the paper.

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